Interference between Resolvable Wavelengths with Single-Photon–Resolved Detection

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(Received 6 October 1987)

Streak-camera detection of moving interference fringes with single-photon resolution constitutes a novel Young's experiment in which the exact frequency of each detected photon cannot be specified without destroying the visibility of the interference pattern. The experiment demonstrates first-order cross spectral coherence between resolvable frequencies, in which case the optical frequency of the detected light must be calculated in terms of a time-dependent spectrum.

PACS numbers: 42.50.Ar, 42.50.Bs

A conventional Young's experiment, conducted with light which is attenuated so that the average time interval between photons is much longer than the transit time through the apparatus,[1] demonstrates, in Dirac's words, that "each photon. . .interferes only with itself." Reconciliation of such wavelike interference phenomena with the photon nature of light is often made by use of Heisenberg's principle of the impossibility of simultaneously measuring the transverse position and momentum of each photon. The question of which pinhole a particular photon came through cannot be answered without destruction of the interference pattern,[2] so that each photon is due to the field modes from both of the pinholes.

Accordingly, in a fully quantized description of Young's interference experiment,[3] the photon intensity at $\mathbf{r},t$ is derived from the sum of field contributions from *both* pinholes, at $\mathbf{r}_1$ and $\mathbf{r}_2$:

$$I(\mathbf{r},t)=\langle\hat{\mathbf{E}}^-(\mathbf{r}_1,t_1)\hat{\mathbf{E}}^+(\mathbf{r}_1,t_1)\rangle+\langle\hat{\mathbf{E}}^-(\mathbf{r}_2,t_2)\hat{\mathbf{E}}^+(\mathbf{r}_2,t_2)\rangle+[\langle\hat{\mathbf{E}}^-(\mathbf{r}_1,t_1)\hat{\mathbf{E}}^+(\mathbf{r}_2,t_2)\rangle+\text{c.c.}]. \quad (1)$$

Here, $\hat{\mathbf{E}}^+$ is the positive-frequency part of the electric field operator, $\hat{\mathbf{E}}^-$ its Hermitean conjugate, and $t_j=t-(|\mathbf{r}-\mathbf{r}_j|)/c$. The last two terms give rise to interference and the fringe visibility depends on the first-order coherence function.

$$G^{(1)}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)=\langle\hat{\mathbf{E}}^-(\mathbf{r}_1,t_1)\hat{\mathbf{E}}^+(\mathbf{r}_2,t_2)\rangle. \quad (2)$$

All expectation values are evaluated with a ket $|\psi\rangle$, which describes the total superposition state of the field. If the total field possesses first-order coherence, it may be expressed as an excitation of just a single field mode: $|\psi\rangle=f(\hat{b}^\dagger)|0\rangle$.[4] Note that this single mode is a linear combination of all the normal modes required to describe the total field.

For example, in interference experiments recently performed with a single-photon number state,[5] we have $|\psi\rangle=\hat{b}^\dagger|0\rangle$, with creation operator

$$\hat{b}^\dagger=(\hat{a}^\dagger_{\mathbf{k}_1}+\hat{a}^\dagger_{\mathbf{k}_2})/\sqrt{2}, \quad (3)$$

where $\mathbf{k}_1$ and $\mathbf{k}_2$ are wave vectors for plane-wave modes from each pinhole with the same polarization. This is to be contrasted with experiments with a highly attenuated coherent state, described by

$$|\psi\rangle=\exp(\alpha\hat{b}^\dagger+\alpha^*\hat{b})|0\rangle, \quad (4)$$

for which the photon number is not precisely defined. In this case, the same results are obtained by a superposition of independently derived coherent beams, since

$$\begin{aligned}|\psi\rangle&=|\alpha_{\mathbf{k}_1}\rangle|\alpha_{\mathbf{k}_2}\rangle\\&=\exp(\alpha_{\mathbf{k}_1}\hat{a}^\dagger_{\mathbf{k}_1}+\alpha^*_{\mathbf{k}_1}\hat{a}_{\mathbf{k}_1})|0\rangle\exp(\alpha_{\mathbf{k}_2}\hat{a}^\dagger_{\mathbf{k}_2}+\alpha^*_{\mathbf{k}_2}\hat{a}_{\mathbf{k}_2})|0\rangle\\&=\exp(\alpha\hat{b}'^\dagger+\alpha^*\hat{b}')|0\rangle,\end{aligned} \quad (5)$$

where

$$\hat{b}'^\dagger=(\alpha_{\mathbf{k}_1}\hat{a}^\dagger_{\mathbf{k}_1}+\alpha_{\mathbf{k}_2}\hat{a}^\dagger_{\mathbf{k}_2})/\alpha. \quad (6)$$

Interference fringes have indeed been detected from the superposition of two independently derived laser beams of essentially the same frequency.[6] Each of the detected photons has an annihilation operator $\hat{b}'$ which is a combination of annihilation operators for wave vectors with different directions: The transverse momentum of the detected photons therefore cannot be specified better than $\hbar(|\mathbf{k}_1-\mathbf{k}_2|)$, if they are to contain contributions from each beam.

For the superposition of coherent sources of different frequencies, the first-order coherence from Eqs. (2) and (5) is

$$G^{(1)}\propto\alpha^*_{\mathbf{k}_1}\alpha_{\mathbf{k}_2}\exp[-i(\mathbf{k}_1\cdot\mathbf{r}_1-\mathbf{k}_2\cdot\mathbf{r}_2)+i(\omega_{\mathbf{k}_1}t_1-\omega_{\mathbf{k}_2}t_2)]. \quad (7)$$

The temporal and spatial dependence is approximated by

$$f(\mathbf{r},t)\approx\exp[-i(\bar{\mathbf{k}}\cdot\Delta\mathbf{r}+\Delta\mathbf{k}\cdot\mathbf{r}-\bar{\omega}\Delta t-\Delta\omega t)], \quad (8)$$

where

$$\bar{\mathbf{k}}=(\mathbf{k}_1+\mathbf{k}_2)/2,\quad \Delta\mathbf{k}=\mathbf{k}_1-\mathbf{k}_2,$$
$$\bar{\omega}=(\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2})/2,\quad \Delta\omega=\omega_{\mathbf{k}_1}-\omega_{\mathbf{k}_2}, \tag{9}$$

and

$$\Delta\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \Delta t=t_1-t_2. \tag{10}$$

This holds for $\Delta\mathbf{k}$ perpendicular to $(\mathbf{r}-\mathbf{r}_1+\mathbf{r}-\mathbf{r}_2)$, such as in the case of beams of slightly different frequencies incident at small angles (i.e., $|\Delta\mathbf{k}|\ll|\bar{\mathbf{k}}|$), for points $\mathbf{r}$ giving small path-length differences between the beams (i.e., $|\mathbf{r}-\mathbf{r}_1|\approx|r-\mathbf{r}_2|$). The detected photon intensity then consists of moving interference fringes described by

$$I(\mathbf{r},t)\propto|\alpha_{\mathbf{k}_1}|^2+|\alpha_{\mathbf{k}_2}|^2+2|\alpha_{\mathbf{k}_1}||\alpha_{\mathbf{k}_2}|\cos(\bar{\mathbf{k}}\cdot\Delta\mathbf{r}+\Delta\mathbf{k}\cdot\mathbf{r}-\bar{\omega}\Delta t-\Delta\omega t+\Phi), \tag{11}$$

where $\Phi=\arg(\alpha_{\mathbf{k}_1})-\arg(\alpha_{\mathbf{k}_2})$. Each of the photons detected in the moving fringes has an annihilation operator $\hat{b}'$ which acts on a combination of modes of unequal frequencies. In addition to lack of specification of the transverse momentum of the detected photons, their energy, $\hbar|\mathbf{k}|/c$, similarly cannot be exactly specified, if they are to contain contributions from each beam.

First-order interference effects between sources of unequal frequencies have previously been demonstrated by analog photodetection of temporal beats from independent laser beams,[7] and even from independent incoherent sources.[8] Beats have been observed with single-photon–resolved detection, with use of a highly attenuated laser beam and a Michelson interferometer with a moving mirror in one arm, which gives a Doppler shift of this component, although by less than the laser linewidth.[9] Quantum beats from a frequency-split fluorescence transition may similarly be attributed to first-order coherence between the different frequency components of each photon. However, in order to observe moving spatial fringes, the two frequency components must originate from spatially separated sources,[10] and a detector which gives both spatial and temporal resolution must be used.

This Letter reports the observation of the first-order coherence, with single-photon–resolved detection, as a function of both space and time, for two spatially separated, wavelength-resolvable beams. This constitutes a Young's experiment in which, if visible fringes are to be obtained, each of the detected photons *must* have its energy as well as transverse momentum unspecified at least to the extent of the difference between the sources.

The two frequency-resolvable beams are produced by passage of the beam from a frequency-stabilized dye laser (linewidth $\simeq$1 MHz) through a 15.25-MHz Bragg cell. This consists of a water-filled cell driven by an accousto-optic quartz crystal and provides close to equal power in the fundamental and Bragg-shifted beams.

The two beams are demonstrated to be of resolvable wavelengths, by combination of them in a collinear beam and measurement of the transmission through a fixed Fabry-Perot etalon as the dye-laser frequency is scanned. The etalon is constructed in a half-confocal arrangement to give calculated intensity finesse of 25 and free spectral range of 148 MHz resulting in a resolution of 6 MHz. Figure 1 shows a scan over about 200 MHz in which two peaks, one from each beam, are clearly resolved, with a measured separation of 15.5 MHz. Additional nonfundamental mode peaks are due to the imperfect alignment and mode coupling into the interferometer, and the measured linewidth of 8.4 MHz indicates an experimental finesse of only about eighteen.

The two beams from the Bragg cell are crossed at a small angle θ, to produce interference fringes orthogonal to the sagittal plane, with a spacing, obtained from the $\Delta\mathbf{k}\cdot\mathbf{r}$ term of Eq. (11), of $\Delta=\pi/|\bar{\mathbf{k}}|\sin(\theta/2)$. The fringes move transverse to the optic axis with a velocity $v=\Lambda\Delta\omega/2\pi$, where $\Delta\omega/2\pi$ is the 15.25-MHz Bragg cell frequency.

The moving fringes are imaged onto the input slit of a streak camera with fringes aligned at right angles to the slit so that they move along the slit and as the streak camera sweeps, diagonal stripes are formed at the streak-camera phosphor. The moving interference fringes were previously set up for a proposed laser-fluorescence velocimeter[11] with moderate-power beams ($\simeq$10 mW), and the streak camera was used to determine their visibility and transverse velocity[12] with use of a low intensifier gain which provides only analog light detection.

By use of maximum gain on the internal microchannel-plate intensifier ($\simeq$3000) and gating to minimize dark noise, a single photon is detectable at the streak-camera photocathode so long as the weak output phos-

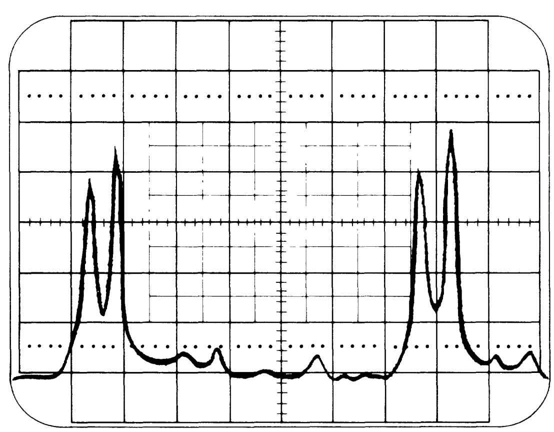


FIG. 1. Intensity transmission of etalon vs dye-laser wavelength.

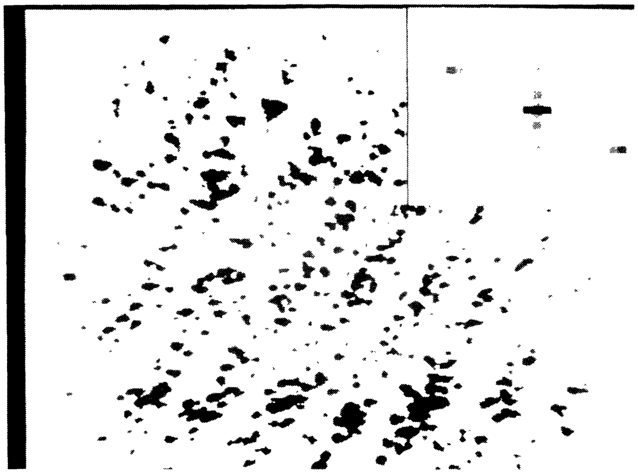


FIG. 2. Superimposed images of single-photon–resolved moving fringes. Streak-camera slit dimension, horizontal, vs time, vertical. Inset: Two-dimensional Fourier transform.

phorescence arising from about 3000 secondary electrons is detectable. This is made possible by reading of the streak-camera phosphor with a high-gain, low-dark-noise camera system, consisting of a microchannel-plate image intensifier and silicon intensified target vidicon. Single spots in the vidicon image are identified as arising from single photons at the streak-camera input slit, by showing that the probability for one's obtaining two photons within an area on the image the size of a spot is negligible, with use of Poisson statistics with the mean number of photons expected from the measured beam intensity.[13]

The beams are attenuated with neutral density filters to a power of about 5 μW each, giving a detection rate of about one photon for each 300-ns sweep, through the 10-μm×5-mm streak-camera slit. In order to collect adequate single-photon spots to recognize the presence of interference fringes, the streak camera is operated in a multisweep mode, by synchronization to the Bragg-cell frequency so that successive images are correctly overlaid.

An image obtained from about 4000 streak-camera sweeps shows spots corresponding to single photons which are individually resolvable. Figure 2 shows the sum of nine such images resulting in identifiable diagonal stripes. The presence of a sinusoidal intensity modulation in both the vertical (time) and horizontal (streak-camera slit) dimensions of the image is clearly apparent from the two distinct off-center peaks in the two-dimensional discrete Fourier transform of the digitized image, shown in the inset.

The presence of interference fringes demonstrates that each detected photon is due to both beams and so must have unspecified transverse momentum. In addition, since the beams are of resolvable frequencies, each photon *must* have its frequency unspecified, at least by this resolution, if it is to originate from both beams. That is, there must be broad spectral response, consistent with the inequality

$$\Delta E \Delta t \geq \hbar, \tag{12}$$

in order to resolve the moving fringes temporally.

In nonrelativistic quantum mechanics, Eq. (12) has a physical interpretation different from that of the relation between conjugate dynamical variables, because time is just a parameter.[14] However, in the quantum theory of the radiation field, position, momentum, and energy are similarly demoted to the status of parameters, so that the uncertainty relations may all be classically derived.[15] Thus Eq. (12) describes the reciprocal relation between the frequency bandwidth and temporal width of a classical wave packet: The spectral profile is the Fourier transform of the temporal intensity profile.

For the Young's experiment described here, a Fourier transform of the first-order coherence function[16] evaluated with Eq. (7) gives

$$W(\mathbf{r}_1,\omega;\mathbf{r}_2,\omega') \propto |\alpha|^2 \exp[-i(\mathbf{k}_1\cdot\mathbf{r}_1-\mathbf{k}_2\cdot\mathbf{r}_2)]\delta(\omega-\omega_{\mathbf{k}_1})\delta(\omega'-\omega_{\mathbf{k}_2}), \tag{13}$$

whereas for a stationary process

$$W(\mathbf{r}_1,\omega;\mathbf{r}_2,\omega') = \tilde{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)\delta(\omega-\omega'), \tag{14}$$

where $\tilde{W}(\mathbf{r}_1,\mathbf{r}_2,\omega)$ is referred to as the cross spectral density. Since Eq. (13) is nonzero for $\omega\neq\omega'$, the experiment may be considered a demonstration of nonzero cross spectral coherence between resolvable frequencies.

Since Eq. (14) does not hold in this experiment, it is clear that the superposition field detected by the streak camera is not stationary and so the question of the frequency spectrum of the detected photons must be addressed with reference to a time-dependent physical spectrum,[17] which may change with time but which depends on how the spectrum is measured. For a Lorentzian measurement with half width Γ, we have

$$S(\omega,t,\Gamma) = 2\Gamma\int_{-\infty}^{t} dt' \int_{-\infty}^{t} dt'' \exp[-\Gamma(2t-t'-t'')-i\omega(t'-t'')]\langle \hat{\mathbf{E}}^{-}(t')\hat{\mathbf{E}}^{+}(t'')\rangle. \tag{15}$$

After substitution of expressions for the time dependence of the electric field operators, derived as the contributions from both beams, the integrals are evaluated to give the optical spectrum at some point $\mathbf{r}$ as[18]

$$S(\omega,t,\Gamma) \propto 2\Gamma[A_1^2+A_2^2+2A_1A_2\cos(\Delta\omega t+\phi)]/A_1^2A_2^2, \tag{16}$$

where $A_j^2 = \Gamma^2 + (\omega - \omega_{k_j})^2$. This indicates temporal oscillations at the beat frequency $\Delta\omega$. If $\Gamma \gg \Delta\omega/2$, the spectral intensity profile consists of a Lorentzian centered at $\bar{\omega}$ with half width Γ, but as Γ is decreased below $\Delta\omega/2$, the magnitude of the temporal oscillations decreases and the two frequencies $\omega_{\mathbf{k}_1}$ and $\omega_{\mathbf{k}_2}$ become resolved. This behavior is consistent with the uncertainty relation of Eq. (12).

The experiment was supported in part by the U.S. Army Research Office under Contract No. DAAG-29-83-K-0110. I also thank C. Parigger, W. Ruyten, L. Smith, D. Keefer, J. Hornkohl, and N. Wright for stimulating discussions and assistance with instrumentation.

[1]G. I. Taylor, Proc. Cambridge Philos. Soc. **15**, 114 (1909); L. Jánossy and Zs. Náray, Acta Phys. Acad. Sci. Hung. **7**, 403 (1957), and Nuovo Cimento, Suppl. **9**, 588 (1958).

[2]J. A. Wheeler, in *Mathematical Foundations of Quantum Theory,* edited by A. R. Marlow (Academic, New York, 1978), p. 9.

[3]L. Mandel, Phys. Rev. **134**, A10 (1964); T. F. Jordan and F. Ghielmetti, Phys. Rev. Lett. **12**, 607 (1964).

[4]U. M. Titulaer and R. J. Glauber, Phys. Rev. **145**, 1041 (1966); D. F. Walls, Am. J. Phys. **45**, 952 (1977).

[5]P. Grangier, G. Roger, and A. Aspect, Europhys. Lett. **1**, 173 (1986); for realization of a one-photon state, see also C. K. Hong and L. Mandel, Phys. Rev. Lett. **56**, 58 (1986).

[6]G. Magar and L. Mandel, Nature (London) **198**, 255 (1963): R. L. Pfleegor and L. Mandel, Phys. Rev. **159**, 1084 (1967), and J. Opt. Soc. Am **58**, 946 (1968); L. Mandel, in *Quantum Optics,* edited by R. L. Glauber (Academic, New York, 1969), p. 176; H. Paul, Rev. Mod. Phys. **58**, 209 (1986).

[7]A. Javan, E. A. Ballik, and W. L. Bond, J. Opt. Soc. Am **52**, 96 (1962).

[8]A. T. Forrester, R. A. Gudmundsen, and P. O. Johnson, Phys. Rev. **99**, 1691 (1955).

[9]C. C. Davis, IEEE J. Quantum Electron. **15**, 26 (1979).

[10]For the case of quantum beats, see P. Grangier, A. Aspect, and J. Vigue, Phys. Rev. Lett. **54**, 418 (1985).

[11]D. R. Keefer, Appl. Opt. **26**, 91 (1987).

[12]L. M. Davis, L. M. Smith, and D. R. Keefer, in "High Speed Photography, Videography and Photonics V," edited by H. C. Johnson, SPIE Proceedings Vol. 832 (International Society for Optical Engineering, Bellingham, WA, to be published).

[13]L. M. Davis, Optical Society of America Annual Meeting, Rochester, New York, 18–23 October 1987 (to be published), paper TuI3.

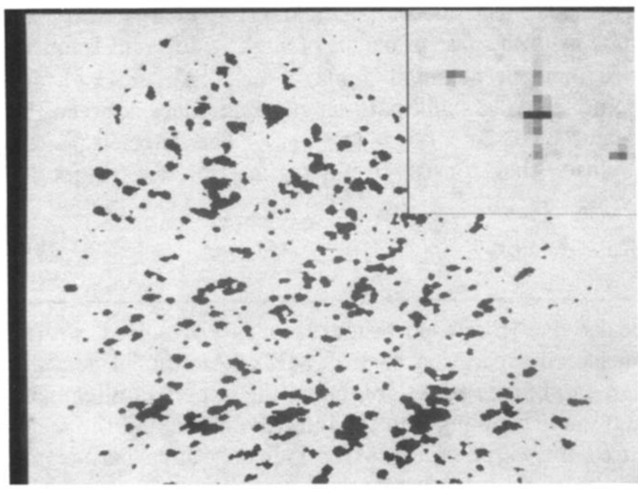
[14]A. Messiah, *Quantum Mechanics* (Wiley, New York, 1958), Vol. 1, p. 135.

[15]G. S. Agrawal, J. T. Foley, and E. Wolf, Opt. Commun. **62**, 67 (1987).

[16]L. Mandel and E. Wolf, J. Opt. Soc. Am. **66**, 529 (1976).

[17]J. H. Eberley and K. Wodkiewicz, J. Opt. Soc. Am. **67**, 1252 (1977); B. Cairns and E. Wolf, Opt. Comm. **62**, 215 (1987).

[18]W. M. Ruyten, L. M. Davis, C. Parigger, and D. R. Keefer, in "Advances in Laser Sciences-III," edited by A. C. Tam, J. O. Gole, and W. C. Stwalley (American Institute of Physics, New York, to be published).



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