EINSTEIN-PODOLSKY-ROSEN PARADOX AND BELL'S INEQUALITY EXPERIMENTS USING TIME AND FREQUENCY

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By using correlations in the signal and idler emissions from a parametric crystal, it should be possible to perform experiments to demonstrate the E—P—R paradox, and to test for violation of a Bell's inequality, using time and frequency measurements.

Violations of Bell's inequality in optics have been largely restricted to experiments utilizing polarization correlations [1]. Recently however, it has been pointed out that phase correlations of parametrically generated photons could demonstrate the E—P—R paradox and furthermore, when heterodyned with a local oscillator, could give rise to a violation of Bell's inequality [2]. This Letter points out the possibility of experiments that would demonstrate the E—P—R paradox and would test for the violation of a Bell's inequality, using correlations in yet another pair of non-compatible variables – time and frequency [3].

Correlations for each of these variables have been previously demonstrated in the emissions from a parametric crystal: signal (1) and idler (2) photons are always emitted at the same time [4] (i.e., \( t^{(1)} = t^{(2)} \)) and energy is conserved [5] (i.e., \( \omega^{(1)} + \omega^{(2)} = \omega^{(\text{pump})} \)). These correlations are quantum mechanically enforced, in that they apply to measurements on two photons produced from the parametric splitting of a single pump photon \(^1\). Theoretical treatments which allow for multimode signal and idler emissions [7] predict that the time correlation is limited by the reciprocal bandwidths of those portions of the downconverted fields which the detectors respond to. Moreover, it is not limited by the coherence time of the pump. Thus, to demonstrate the E—P—R paradox using time and frequency variables, a bandwidth-limited pump pulse would be used to pump the crystal and accurate measurements would be made of both the detection time of photon (1) with respect to the center of the pump pulse, and the spectral frequency of photon (2). Because the detection of time of photon (1) gives knowledge of the arrival time of photon (2), the arrival time and the spectral frequency of photon (2) become known to a precision which violates the time-frequency bandwidth product.

A possible experimental set-up, within present-day technological limitations for measuring the time or frequency of a single photon, is shown in fig. 1. A nominally 10 ns/88 MHz bandwidth-limited pulse would pump the parametric crystal, which would be thin enough to ensure phase matching over the full pump bandwidth. The parametric emissions (1) and (2) would then also be of 10 ns duration, but would extend over a wider bandwidth than 88 MHz, as shown by the dashed line ovals at the bottom of fig. 1. The detection time of a single photon from emission (1) would be measured with 100 ps resolution electronics, and the frequency of a single photon from emission (2) would be measured with a 10 MHz resolution etalon.

To ensure enforcement of correlations, the photons (1) and (2) must originate from the same pump photon. This would be achieved by attenuating the pump pulses so that the average number of photons per pump pulse, \( \langle n \rangle \), is much less than 1. Thus the Poisson probability of two or more photons per pump...
not addressed by Bohr's reply: Measurements on photons (1) and (2) may be space-like separated, so that they cannot be causally linked. This allows detectors (1) and (2) to respond to non-conjugate sets of modes, so that each may measure a parameter independent of the other. Whether such measurements give rise to elements of "physical reality" denotes this time/frequency example of the E—P—R paradox to a philosophical issue, as is the case for any example, including the original [9]. In order to move from philosophy back to physics, we must consider an experiment to test for violation of a Bell's inequality [10].

The result of a Bell's inequality experiment using time and frequency measurements would be of considerable interest, because these variables have a special status in quantum optics [11]: Time is seldom [12] considered as an observable (or even an operator), but is merely a parameter on which other operators may depend. Frequency, which is related to the energy of the photon by Planck's constant, is obtained as the classical Fourier transform of the temporal fluctuations of the electric field, and is to be distinguished from the Hamiltonian operator. It is not considered as a quantum mechanical operator but rather is treated at a classical level by decomposition of the electric field fluctuations into Fourier-orthogonal modes. These are then quantized in the same manner as a harmonic oscillator [13].

It is possible to have a single quantum in a linear combination of such Fourier-orthogonal modes. For example, Dirac's explanation of first order interference in Young's two-slit experiment [14] as being due to "each photon ... interfering only with itself", attributes each detected photon to a linear combination of the Fourier-orthogonal modes from the two slits [15] \(^2\). The modes may have not only different momentum directions, \(k_1\) and \(k_2\), but also different frequencies, \(\omega_1\) and \(\omega_2\), as verified in an experimental demonstration of interference between resolvable frequencies at single photon levels [17]. Although that experiment was conducted with a highly atten-

\(^2\) Note that while all fields which possess first order optical coherence can be described as a "single mode" excitation [16] this "single mode" is a linear combination of all the Fourier-orthogonal modes, with different wavevectors, required to describe the total field.
uated coherent state given in six basic steps below.

(1) In addition to the two non-compatible observables \( \hat{S}_x, \hat{S}_y \), choose a third observable which partially specifies both \( \hat{S}_x = \cos \theta \hat{S}_x + \sin \theta \hat{S}_y \). One measurement which partially specifies both the time \( \hat{T} \) and the frequency \( \hat{F} \) of a light signal is that of the time-dependent-physical-spectrum \( \hat{F} \) (TDPS or \( \hat{T} \)). A TDPS measurement can be made with single photon resolution, by using a fast single photon detector behind a Fabry—Perot etalon, in such a way that the photon arrival time is measured to a precision limited only by the reciprocal of the etalon frequency resolution. Thus the quality-factor of the etalon, \( Q \), determines whether one measures time, \( \hat{T} \), \( Q = 0 \), or frequency, \( \hat{F} \), \( Q = \infty \), or some intermediate combination of frequency and time, \( \hat{F}_Q \), \( 0 < Q < \infty \).

(2) For variables which may take a continuous range of values, use some form of “windowing” to obtain a discretized spectrum with only two eigenvalues. For the frequency measurement, \( \hat{F} \), (or spin measurement \( \hat{S}_z \)) such windowing is automatically imposed by the etalon pass-band (Stern—Gerlach analyzer), so that either plus (+) for transmission or minus (−) for reflection results. For measurements of \( \hat{T} \) or \( \hat{F} \), windowing may be added \( [23] \) by using a single-channel-analyzer with the time-measuring electronics, to select photons detected within a preset time window. Thus \( \hat{F}_Q \) has eigenvalue + if the photon is passed by the etalon and is detected within this time window.

(3) Identify a correlation between the discrete results of measurements on two space-like separated quanta \( (\hat{S}^{(1)}_x = +) \Leftrightarrow (\hat{S}^{(2)}_x = -) \). For measurements on conjugate photons of \( \hat{F} \) with spectral and temporal windows centered on the emissions, the conservation rules \( t^{(1)} = t^{(2)} \) and \( \omega^{(1)} + \omega^{(2)} = \omega^{(\text{pump})} \) should give rise to the direct correlations:

\[
\hat{F}^{(1)}_Q = + \Leftrightarrow \hat{F}^{(2)}_Q = +, \quad \hat{F}^{(1)}_Q = - \Leftrightarrow \hat{F}^{(2)}_Q = - .
\]

These correlations, which are expected from the theoretical treatments of ref. [7], could be experimentally verified using space-like separated measurements. If “contra-factual definiteness” (cfd) is assumed (i.e., \( \hat{T}, \hat{F} \) and \( \hat{P} \) all take definite values for each quantum, irrespective of which, if any, is mea-
a) Correlations:

\[
\begin{align*}
\text{Time} & \quad Q = \infty \\
\text{Frequency} & \quad (1) \quad (2) \\
\text{Correlations} & \quad \hat{F}_+ c_2 \hat{F}_- \\
\end{align*}
\]

b) Bell's Inequality:

\[
\begin{align*}
\hat{F}_+ & \oplus \hat{T}_+ \oplus \hat{F}_- \oplus \hat{T}_- \\
\leq & \quad \hat{F}_+ \oplus \hat{T}_+ \oplus \hat{F}_- \oplus \hat{T}_- \\
\end{align*}
\]

Fig. 2. (a) Expected correlations in measurements of the “windowed” time-dependent spectrum, as in eqs. (4). (b) Bell inequality (6), with assumptions of eqs. (4) and (5).

... assumed), the correlations can be illustrated by fig. 2a.

(4) Measure three sets of coincidence count-rates \( R \), where the coincidence is taken to be within the reciprocal of the laser repetition rate (and the frequency measurements \( \hat{F} \) are made to a resolution no better than this). These are \( R[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] \), \( R[\hat{F}^{(1)} = +, \hat{F}^{(2)} = -] \), and \( R[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] \) (eqs. (3.9.7) and (3.9.8) of ref. [19]).

(5) Use the assumptions of cfd and of Einstein locality (i.e., that measurements on one quantum cannot affect the results of measurements on the other since the measurements are space-like separated) to obtain independence of each measurement of the pair, and thus derive the following probabilities:

\[
\begin{align*}
\text{Pr}[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] & = \text{Pr}[\hat{F}^{(1)} = +] \text{ Pr}[\hat{T}^{(2)} = -] , \\
\text{Pr}[\hat{F}^{(1)} = +, \hat{F}^{(2)} = -] & = \text{Pr}[\hat{F}^{(1)} = +] \text{ Pr}[\hat{F}^{(2)} = -] , \\
\text{Pr}[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] & = \text{Pr}[\hat{F}^{(1)} = +] \text{ Pr}[\hat{T}^{(2)} = -] . \\
\end{align*}
\]

(6) Use the direct correlations of eqs. (4) to equate these probabilities with

\[
\begin{align*}
\text{Pr}[\hat{F}^{(1)} = +] \text{ Pr}[\hat{F}^{(1)} = -] , \\
\text{Pr}[\hat{F}^{(1)} = +] \text{ Pr}[\hat{F}^{(1)} = -] , \\
\text{Pr}[\hat{F}^{(1)} = +] \text{ Pr}[\hat{F}^{(1)} = -] , \\
\end{align*}
\]

respectively. This results in the inequality illustrated in fig. 2b:

\[
\begin{align*}
\text{Pr}[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] & \leq \text{Pr}[\hat{F}^{(1)} = +, \hat{F}^{(2)} = -] \\
& + \text{Pr}[\hat{F}^{(1)} = +, \hat{T}^{(2)} = -] . \\
\end{align*}
\]

Whether such an inequality can be violated should ultimately be experimentally decided, and it is the key purpose of this Letter to propose this novel type of Bell's inequality experiment. If, as previously suggested, \( \hat{T}, \hat{F} \) and \( \hat{F} \) measurements on single photons do indeed behave as quantum mechanical projections, they would have an operator algebra similar to that of the spin operators \( \hat{S}_x, \hat{S}_y \) and \( \hat{S}_z \). Hence we might expect eqs. (5) to be invalid and inequality (6) to be violated (in parallel with pp. 229–232 of ref. [19]).

Just the original Bell inequality was modified into a form more amenable for experimental testing [24], inequality (6) would also need to be modified. Experimental proof of a violation is technically difficult because of the imperfect quantum efficiencies of the photon detectors and throughout of the etalons, so that an additional “no-enhancement” assumption [25] may be necessary. However, present-day time and frequency resolution capabilities for single photon detection are clearly adequate for a feasible experiment.

Also, it may be possible to derive a time–frequency Bell inequality without the cfd assumption [26], and chaining [27] may yield an inequality which could be more strongly violated. Lastly, this example of Bell's inequality would pose an interesting example for comparing the increasingly popular transactional interpretation [28] with the standard Copenhagen interpretation of quantum formalism.

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References


