EINSTEIN–PODOLSKY–ROSEN PARADOX AND BELL'S INEQUALITY EXPERIMENTS USING TIME AND FREQUENCY

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By using correlations in the signal and idler emissions from a parametric crystal, it should be possible to perform experiments to demonstrate the E–P–R paradox, and to test for violation of a Bell's inequality, using time and frequency measurements.

Violations of Bell's inequality in optics have been largely restricted to experiments utilizing polarization correlations [1]. Recently however, it has been pointed out that phase correlations of parametrically generated photons could demonstrate the E–P–R paradox and furthermore, when heterodyned with a local oscillator, could give rise to a violation of Bell's inequality [2]. This Letter points out the possibility of experiments that would demonstrate the E–P–R paradox and would test for the violation of a Bell's inequality, using correlations in yet another pair of non-compatible variables – time and frequency [3].

Correlations for each of these variables have been previously demonstrated in the emissions from a parametric crystal: signal (1) and idler (2) photons are always emitted at the same time [4] (i.e., $t^{(1)}=t^{(2)}$) and energy is conserved [5] (i.e., $\omega^{(1)}+\omega^{(2)}=\omega^{(\text{pump})}$). These correlations are quantum mechanically enforced, in that they apply to measurements on two photons produced from the parametric splitting of a single pump photon [#1]. Theoretical treatments which allow for multimode signal and idler emissions [7] predict that the time correlation is limited by the reciprocal bandwidths of those portions of the downconverted fields which the detectors respond to. Moreover, it is not limited by the coherence time of the pump. Thus, to demonstrate the E–P–R paradox using time and frequency variables, a bandwidth-limited pump pulse would be used to pump the crystal and accurate measurements would be made of both the detection time of photon (1) with respect to the center of the pump pulse, and the spectral frequency of photon (2). Because the detection of time of photon (1) gives knowledge of the arrival time of photon (2), the arrival time and the spectral frequency of photon (2) become known to a precision which violates the time-frequency bandwidth product.

A possible experimental set-up, within present-day technological limitations for measuring the time or frequency of a single photon, is shown in fig. 1. A nominally 10 ns/88 MHz bandwidth-limited pulse would pump the parametric crystal, which would be thin enough to ensure phase matching over the full pump bandwidth. The parametric emissions (1) and (2) would then also be of 10 ns duration, but would extend over a wider bandwidth than 88 MHz, as shown by the dashed line ovals at the bottom of fig. 1. The detection time of a single photon from emission (1) would be measured with 100 ps resolution electronics, and the frequency of a single photon from emission (2) would be measured with a 10 MHz resolution etalon.

To ensure enforcement of correlations, the photons (1) and (2) must originate from the same pump photon. This would be achieved by attenuating the pump pulses so that the average number of photons per pump pulse, $\langle n\rangle$, is much less than 1. Thus the Poisson probability of two or more photons per pump

[#1] Multi-photon states can also exhibit correlations leading to a violation of Bell's inequality; see ref. [6].

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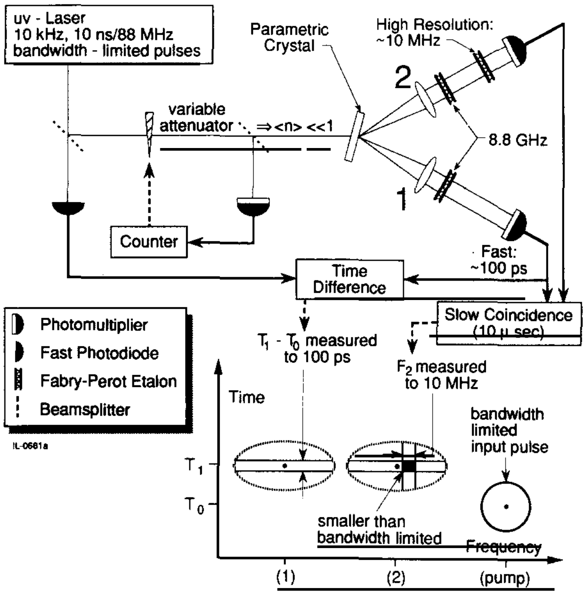

Fig. 1. Possible E–P–R experiment using time and frequency measurements on parametrically generated photons.

pulse, which would give rise to accidental coincidences, is kept less than the probability of truly "conjugate" pairs of photons (1) and (2), as calculated from the slow coincidence count rate.

Note however, that detectors (1) and (2) do *not* respond to equivalent (i.e., "conjugate") sets of modes, since the 10 MHz etalon restricts the modes seen by detector (2). This may seem to restrict the applicability of the theoretical models of ref. [7], which only treat the case in which the sets of modes seen by detectors (1) and (2) are conjugate. However, this case is all that is necessary if one agrees with Bohr's "complementarity" reply to the E–P–R paradox [8]. For Bohr's reply reaffirms the positivist attitude to reality given by the Copenhagen interpretation, in that when the frequency of photon (2) is measured (by restricting the modes which detector (2) responds to), "the physical reality" of the time parameter for this photon is compromised. Moreover, according to Bohr, the total physical system must be considered, so that "the physical reality" of the time parameter for photon (1) is also compromised, thereby similarly restricting the modes which detector (1) responds to.

According to E–P–R however, the central issue is not addressed by Bohr's reply: Measurements on photons (1) and (2) may be space-like separated, so that they cannot be causally linked. This allows detectors (1) and (2) to respond to non-conjugate sets of modes, so that each may measure a parameter independent of the other. Whether such measurements give rise to elements of "physical reality" denotes this time/frequency example of the E–P–R paradox to a philosophical issue, as is the case for any example, including the original [9]. In order to move from philosophy back to physics, we must consider an experiment to test for *violation of a Bell's inequality* [10].

The result of a Bell's inequality experiment using time and frequency measurements would be of considerable interest, because these variables have a special status in quantum optics [11]: Time is seldom [12] considered as an observable (or even an operator), but is merely a parameter on which other operators may depend. Frequency, which is related to the energy of the photon by Planck's constant, is obtained as the classical Fourier transform of the temporal fluctuations of the electric field, and is to be distinguished from the Hamiltonian operator. It is not considered as a quantum mechanical operator but rather is treated at a classical level by decomposition of the electric field fluctuations into Fourier-orthogonal modes. These are then quantized in the same manner as a harmonic oscillator [13].

It is possible to have a single quantum in a linear combination of such Fourier-orthogonal modes. For example, Dirac's explanation of first order interference in Young's two-slit experiment [14] as being due to "each photon ... interfering only with itself", attributes each detected photon to a linear combination of the Fourier-orthogonal modes from the two slits [15] [#2]. The modes may have not only different momentum directions, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, but also different frequencies, ω_1 and ω_2, as verified in an experimental demonstration of interference between resolvable frequencies at single photon levels [17]. Although that experiment was conducted with a highly atten-

[#2] Note that while all fields which possess first order optical coherence can be described as a "single mode" excitation [16] this "single mode" is a linear combination of all the Fourier-orthogonal modes, with different wavevectors, required to describe the total field.

uated coherent state

$$|\psi\rangle = \exp(\alpha \hat{b}^\dagger + \alpha^* \hat{b})|0\rangle\,, \quad |\alpha| \ll 1\,, \tag{1}$$

where

$$\hat{b}^\dagger = (\hat{a}^\dagger_{k_1} + \hat{a}^\dagger_{k_2})/\sqrt{2}\,, \tag{2}$$

first order interference between two frequency components should also occur with the number state

$$|\psi\rangle = \hat{b}^\dagger|0\rangle\,. \tag{3}$$

Indeed, quantum beats from a frequency-split atomic transition [18] may be attributed to two-frequency single-photon emission from each atom. If such a bichromatic one-photon Fock state, as described in eq. (3), were incident upon a high resolution frequency-filter tuned to one of the two frequency components, say ω_1, it is expected to behave quantum mechanically in that either a whole photon is detected at the output of the filter or a whole photon is detected after being reflected from the filter [#3]. After the frequency measurement is made, the detected photon may be ascribed either frequency ω_1 or frequency ω_2, and the state "projects out" to become either $\hat{a}^\dagger_{k_1}|0\rangle$ or $\hat{a}^\dagger_{k_2}|0\rangle$. In this sense the frequency measurement of a single photon behaves in the same quantum mechanical manner as a polarization measurement of a single photon [20], or a spin measurement of a single fermion performed with a Stern–Gerlach analyzer [21].

It is also possible to conceptualize a time measurement on a double-peaked one-photon wavepacket such that a whole photon is detected either in the first peak, or else in the second. In this sense, time also appears to behave as a quantum mechanical operator. If a Bell's inequality using time and frequency measurements is found to be violated, this could be taken as further reason to elevate these variables to the status of quantum mechanical operators.

A Bell's inequality for time and frequency may be obtained by following Sakurai's pedagogical derivation for spin operators (section 3.9 of ref. [19]), given in six basic steps below.

(1) In addition to the two non-compatible observables $(\hat{S}_x, \hat{S}_y)$, choose a third observable which partially specifies both $\hat{S}_\theta = \cos\theta\,\hat{S}_x + \sin\theta\,\hat{S}_y$). One measurement which partially specifies both the time $(\hat{T})$ and the frequency $(\hat{F})$ of a light signal is that of the time-dependent-physical-spectrum [22] (TDPS or $\hat{\Gamma}$). A TDPS measurement can be made with single photon resolution, by using a fast single photon detector behind a Fabry–Perot etalon, in such a way that the photon arrival time is measured to a precision limited only by the reciprocal of the etalon frequency resolution. Thus the quality-factor of the etalon, Q, determines whether one measures time, $\hat{T}$, $(Q=0)$, or frequency, $\hat{F}$, $(Q=\infty)$, or some intermediate combination of frequency and time, $\hat{\Gamma}_Q$, $(0<Q<\infty)$.

(2) For variables which may take a continuous range of values, use some form of "windowing" to obtain a discretized spectrum with only two eigenvalues. For the frequency measurement, $\hat{F}$, (or spin measurement $\hat{S}_\alpha$) such windowing is automatically imposed by the etalon pass-band (Stern–Gerlach analyzer), so that either plus (+) for transmission or minus (−) for reflection results. For measurements of $\hat{T}$ or $\hat{\Gamma}$, windowing may be added [23] by using a single-channel-analyzer with the time-measuring electronics, to select photons detected within a preset time window. Thus $\hat{\Gamma}_Q$ has eigenvalue + if the photon is passed by the etalon and is detected within this time window.

(3) Identify a correlation between the discrete results of measurements on two space-like separated quanta $(\hat{S}^{(1)}_\alpha = + \Leftrightarrow \hat{S}^{(2)}_\alpha = -)$. For measurements on conjugate photons of $\hat{\Gamma}_Q$ with spectral and temporal windows centered on the emissions, the conservation rules $t^{(1)} = t^{(2)}$ and $\omega^{(1)} + \omega^{(2)} = \omega^{(\mathrm{pump})}$ should give rise to the direct correlations:

$$\hat{\Gamma}^{(1)}_Q = + \Leftrightarrow \hat{\Gamma}^{(2)}_Q = +\,, \quad \hat{\Gamma}^{(1)}_Q = - \Leftrightarrow \hat{\Gamma}^{(2)}_Q = -\,. \tag{4}$$

These correlations, which are expected from the theoretical treatments of ref. [7], could be experimentally verified using space-like separated measurements. If "contra-factual definiteness" (cfd) is assumed (i.e., $\hat{T}$, $\hat{F}$ and $\hat{\Gamma}$ all take definite values for each quantum, irrespective of which, if any, is mea-

[#3] This predicted behavior has not been experimentally verified for a bichromatic one photon Fock state, but has been observed for the highly attenuated bichromatic coherent field of eq. (1) in an extension of the experiment of ref. [14] (see ref. [19]).

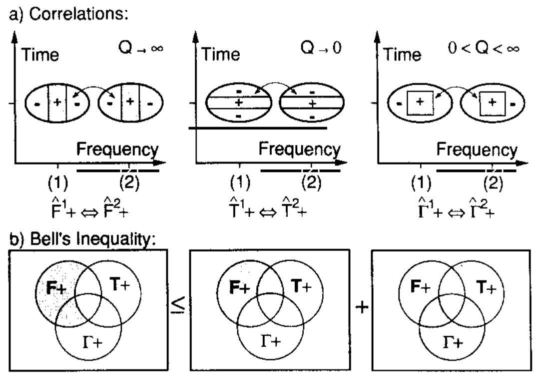

Fig. 2. (a) Expected correlations in measurements of the "windowed" time-dependent spectrum, as in eqs. (4). (b) Bell inequality (6), with assumptions of eqs. (4) and (5).

sured), the correlations can be illustrated by fig. 2a.

(4) Measure three sets of coincidence count-rates R, where the coincidence is taken to be within the reciprocal of the laser repetition rate (and the frequency measurements $\hat{F}$ are made to a resolution no better than this). These are $R[\hat{F}^{(1)}=+, \hat{T}^{(2)}=-]$, $R[\hat{F}^{(1)}=+, \hat{\Gamma}^{(2)}=-]$, and $R[\hat{\Gamma}^{(1)}=+, \hat{T}^{(2)}=-]$ (eqs. (3.9.7) and (3.9.8) of ref. [19]).

(5) Use the assumptions of cfd and of Einstein locality (i.e., that measurements on one quantum cannot affect the results of measurements on the other since the measurements are space-like separated) to obtain independence of each measurement of the pair, and thus derive the following probabilities:

$$\begin{aligned}
&\Pr[\hat{F}^{(1)}=+, \hat{T}^{(2)}=-] \\
&\quad=\Pr[\hat{F}^{(1)}=+]\Pr[\hat{T}^{(2)}=-]\,, \\
&\Pr[\hat{F}^{(1)}=+, \hat{\Gamma}^{(2)}=-] \\
&\quad=\Pr[\hat{F}^{(1)}=+]\Pr[\hat{\Gamma}^{(2)}=-]\,, \\
&\Pr[\hat{\Gamma}^{(1)}=+, \hat{T}^{(2)}=-] \\
&\quad=\Pr[\hat{\Gamma}^{(1)}=+]\Pr[\hat{T}^{(2)}=-]\,. \qquad (5)
\end{aligned}$$

(6) Use the direct correlations of eqs. (4) to equate these probabilities with

$$\begin{aligned}
&\Pr[\hat{F}^{(1)}=+]\Pr[\hat{T}^{(1)}=-]\,, \\
&\Pr[\hat{F}^{(1)}=+]\Pr[\hat{\Gamma}^{(1)}=-]\,, \\
&\Pr[\hat{\Gamma}^{(1)}=+]\Pr[\hat{T}^{(1)}=-]\,,
\end{aligned}$$

respectively. This results in the inequality illustrated in fig. 2b:

$$\begin{aligned}
&\Pr[\hat{F}^{(1)}=+, \hat{T}^{(2)}=-] \\
&\quad\leqslant \Pr[\hat{F}^{(1)}=+, \hat{\Gamma}^{(2)}=-] \\
&\quad+\Pr[\hat{\Gamma}^{(1)}=+, \hat{T}^{(2)}=-]\,. \qquad (6)
\end{aligned}$$

Whether such an inequality can be violated should ultimately be experimentally decided, and it is the key purpose of this Letter to propose this novel type of Bell's inequality experiment. If, as previously suggested, $\hat{T}$, $\hat{F}$ and $\hat{\Gamma}$ measurements on single photons do indeed behave as quantum mechanical projections, they would have an operator algebra similar to that of the spin operators $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_\theta$. Hence we might expect eqs. (5) to be invalid and inequality (6) to be violated (in parallel with pp. 229–232 of ref. [19]).

Just the original Bell inequality was modified into a form more amenable for experimental testing [24], inequality (6) would also need to be modified. Experimental proof of a violation is technically difficult because of the imperfect quantum efficiencies of the photon detectors and throughout of the etalons, so that an additional "no-enhancement" assumption [25] may be necessary. However, present-day time and frequency resolution capabilities for single photon detection are clearly adequate for a feasible experiment.

Also, it may be possible to derive a time–frequency Bell inequality without the cfd assumption [26], and chaining [27] may yield an inequality which could be more strongly violated. Lastly, this example of Bell's inequality would pose an interesting example for comparing the increasingly popular transactional interpretation [28] with the standard Copenhagen interpretation of quantum formalism.

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